UniCrypt 2.0

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Outline

Hash Function with Multiple Arguments

Representing Elements by Natural Numbers

Simplified Element Interfaces

Proper RSA Implementation

Nomenclature and Factories

Extensions

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Hash Function with Multiple Arguments

- Hash function $H : \{0,1\}^* \to \{0,1\}^n$
- Collision: $H(x) = H(y)$ for $x \neq y$
- Hash function with multiple arguments:

$$H : \{0,1\}^* \times \cdots \times \{0,1\}^* \to \{0,1\}^n$$

- Collision with multiple arguments:

$$H(x_1,\ldots,x_k) = H(y_1\ldots,y_k),\ \text{for } (x_1,\ldots,x_k) \neq (y_1,\ldots,y_k)$$

- Note that concatenation $|| : \{0,1\}^* \times \cdots \times \{0,1\}^* \to \{0,1\}^*$ leads to collisions, for example:

$$H(0,01) = H(0||01) = H(00||1) = H(00,1)$$

Current Solution: String Concatenation

- Alphabet $\mathcal{A}$ (for example $\mathcal{A} = \{0, 1, \ldots, 9\}$)
- Encoding: $\varepsilon : \{0,1\}^* \to \mathcal{A}^*$
- Separator: $s \notin \mathcal{A}$
- Decoding: $\delta : (\mathcal{A} \cup \{s\})^* \to \{0,1\}^*$
- Hash function with multiple arguments:

$$H(x_1, \ldots, x_k) = H(\delta(\varepsilon(x_1)||s|| \cdots ||s||\varepsilon(x_k)))$$

- If x_i itself is a tuple, it gets even more complicated

New Solution 1: Hash of Hash Values

- Hash function with multiple arguments:

$$H(x_1, \ldots, x_k) = H(H(x_1) || \cdots || H(x_k))$$

- Same collision probability as the hash function itself

New Solution 2: Pairing (Tuple) Function

- Encoding: $\varepsilon : \{0,1\}^* \to \mathbb{N}$
- Pairing (tuple) function: $\psi_k : \mathbb{N}^k \to \mathbb{N}$
- Decoding: $\delta : \mathbb{N} \to \{0,1\}^*$
- Hash function with multiple arguments:

$$H(x_1, \ldots, x_k) = H(\delta(\psi_k(\varepsilon(x_1), \ldots, \varepsilon(x_k))))$$

- If we choose ε and δ to be the standard binary representation, we can write

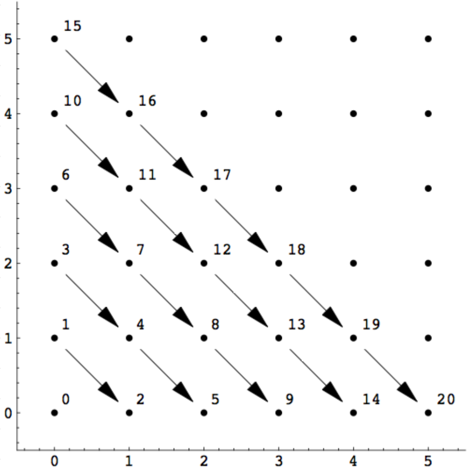
$$H(x_1, \ldots, x_k) = H(\psi_k(x_1, \ldots, x_k))$$

- Note that in UniCrypt, we already work with natural numbers most of the time

Cantor Pairing Function

- A *pairing function* maps two natural numbers bijectively into a single natural number
- The *Cantor pairing function* is defined as follows:
$$\psi(x_1, x_2) = \frac{1}{2}(x_1 + x_2)(x_1 + x_2 + 1) + x_2$$
- Note that $|\psi(x_1, x_2)| = 2 \cdot \max(|x_1|, |x_2|) + 1$
- The inverse function $\psi^{-1} : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ is called *unpairing function*
- For $y = \psi(x_1, x_2)$, let $s = \lfloor \frac{\sqrt{8y+1}-1}{2} \rfloor$ and $t = \frac{1}{2}(s^2 + s)$
- This implies:
$$(x_1, x_2) = \psi^{-1}(y) = (y - t, s + t - y)$$

Cantor Pairing Function



Elegant Pairing Function

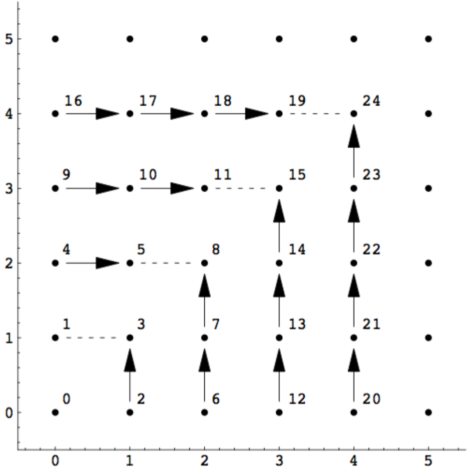
- The *elegant pairing function* is defined as follows:

$$\psi(x_1, x_2) = \begin{cases} x_1^2 + x_1 + x_2, & \text{if } x_1 \geq x_2 \\ x_1 + x_2^2, & \text{otherwise} \end{cases}$$

- Note that $|\psi(x_1, x_2)| = 2 \cdot \max(|x_1|, |x_2|)$
- For $y = \psi(x_1, x_2)$, let $s = \lfloor \sqrt{y} \rfloor$ and $t = y - s^2$
- This implies:

$$(x_1, x_2) = \psi^{-1}(y) = \begin{cases} (t, s), & \text{if } t < s \\ (s, t - s), & \text{otherwise} \end{cases}$$

Elegant Pairing Function



Tuple Function

- Any pairing function can be generalized recursively to a tuple function $\psi_k : \mathbb{N}^k \to \mathbb{N}$:

$$\psi_k(x_1, \ldots, x_k) = \begin{cases} \psi(\psi_{k-1}(x_1, \ldots, x_{k-1}), x_k), & \text{if } k > 2 \\ \psi(x_1, x_2), & \text{if } k = 2 \end{cases}$$

- The problem with this construction is the exponential size of the result (relative to k)
- Improved generalization with linear size (relative to k):

$$\psi_k(x_1, \ldots, x_k) = \begin{cases} \psi_{\frac{k}{2}}(\psi(x_1, x_2), \ldots,), & \text{if } k > 2 \text{ is even} \\ \psi_{\frac{k+1}{2}}(\psi(x_1, x_2), \ldots, x_k), & \text{if } k > 2 \text{ is odd} \\ \psi(x_1, x_2), & \text{if } k = 2 \end{cases}$$

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Groups and Elements

- UniCrypt works with *atomic elements* of *atomic groups*
  - → `ZPlus`: $\mathbb{Z}$
  - → `ZPlusMod`: $\mathbb{Z}_n$
  - → `ZStarMod`: $\mathbb{Z}_n^*$
  - → `GStarMod`, `GStarPrime`, `GStarSave`: $G_q \subseteq \mathbb{Z}_n^*$
  - → `BooleanGroup`: $\mathbb{B} = \{true, false\}$
  - → `SingletonGroup`: $\mathbb{S} = \{\diamond\}$
  - → `PermutationGroup`: Π_k
- and arbitrarily complex *tuple elements* of
  - → `ProductGroup`: $G_1 \times \cdots \times G_k$
  - → `PowerGroup`: G^k

Representing Atomic Elements

- For every atomic group G, there is an injective function
  - → $\phi_G : G \to \mathbb{Z}$, for `ZPlus`
  - → $\phi_G : G \to \mathbb{N}$, for all other groups

  with $\phi_G^{-1}(\phi_G(e)) = e$ for all $e \in G$
- In UniCrypt, $\phi_G(e)$ corresponds to `G.getBigInteger(e)` and $\phi_G^{-1}(x)$ corresponds to `G.createElement(x)`
- No such mapping exists for tuple elements, i.e., for elements of `ProductGroup` or `PowerGroup`
- **Goal**: Representation $\phi_G : G \to \mathbb{N}$ for all possible elements

Represening Elements of `ZPlus`

- Currently, `ZPlus` is needed as a group of infinite order
  - → $(\mathbb{Z}, +)$ is a cyclic group
  - → $(\mathbb{Z}, *)$, $(\mathbb{N}, +)$, $(\mathbb{N}, *)$, ... are not groups
- For an element of `ZPlus`, we can apply any invertible function $F : \mathbb{Z} \to \mathbb{N}$ to its integer representation
- For example, let

$$F(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -(2x+1), & \text{otherwise} \end{cases}$$

- For $y = F(x)$, we get

$$x = F^{-1}(y) = \begin{cases} \frac{1}{2}z, & \text{if } Y \text{ is even} \\ -\frac{1}{2}(y+1), & \text{otherwise} \end{cases}$$

Representing Tuple Elements

- For elements of `ProductGroup` and `PowerGroup`, we can recursively apply the tuple function ψ_k
- Let $(e_1, \ldots, e_k) \in G$ be an element of $G = G_1 \times \cdots \times G_k$, then

  $$\phi_G(e_1, \ldots, e_k) = \psi_k(\phi_{G_1}(e_1), \ldots, \phi_{G_k}(e_k))$$

  is its unique integer representation
- Note that if all group elements can be represented uniquely by a natural number, then we can directly compute the element's hash value

Outline

Elements and Sub-Types

- In UniCrypt, there is a one-to-one correspondence between groups and elements
  - → `Group` ⇔ Element
  - → `AtomicGroup` ⇔ AtomicElement
  - → `AdditiveGroup` ⇔ AdditiveElement
  - → `MultiplicativeGroup` ⇔ MultiplicativeElement
  - → `BooleanGroup` ⇔ BooleanElement
  - → `ProductGroup`, `PowerGroup` ⇔ TupleElement
  - → `PermutationGroup` ⇔ PermutationElement
- Trivial type casts are often necessary to guarantee this correspondence
- Generics seems to be a natural solution, but it only works in one direction

Proposal for UniCrypt 2.0

- Reduce everything to `Element`:
  - → `AtomicElement` is no longer needed, since all elements map to natural numbers (see previous section)
  - → `AdditiveElement` only adds syntactic sugar for writing `e1.add(e2)` instead of `e1.apply(e2)`
  - → `MultiplicativeElement` only adds syntactic sugar for writing `e1.multiply(e2)` instead of `e1.apply(e2)`
  - → `BooleanElement` only adds syntactic sugar for writing `true/false` instead of `1/0`
  - → `TupleElement` allows accessing the i-the component of a tuple element of arity k, but this includes the 'atomic' case of $k = 1$
- `PermutationElement` seems to be the only true specialization of `Element`

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Algebraic View of RSA

- RSA does not work with groups
  - → $n = p{\cdot}q$
  - → $e \in \mathbb{Z}^*_{\phi(n)}$
  - → $d = e^{-1} \bmod \phi(n)$
  - → $m \in \mathbb{Z}_n$
  - → $c = Enc_e(m) = m^e \bmod n \in \mathbb{Z}_n$
- $Enc_e : \mathbb{Z}_n \to \mathbb{Z}_n$, but $(\mathbb{Z}_n, *)$ is not a group
- However, $(\mathbb{Z}_n, *)$ is a *monoid* (group without invertibility)
- Therefore, to implement RSA properly in UniCrypt, we need a super-type `Monoid` of `Group`
- Note that $(\mathbb{N}, +)$ is also a monoid: `NPlus` instead of `ZPlus`?

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UniCrypt Nomenclature

- The UniCrypt nomenclature for interface and class names is very self-consistent:
  - → `class UserClass implements User`
  - → `class UserAbstract implements User`
- Other naming styles:
  - → `class User implements IUser`
  - → `class CUser implements IUser`
  - → `class UserImpl implements User`
  - → `class DefaultUser implements User`
  - → `class AbstractUser implements User`

Suggestions and Solutions

- "Name your interface what it is."
  - → If your interface is a *truck*, call it `Truck` (not `ITruck` because it isn't an *itruck*)
  - → Then write implementations `DumpTruck`, `CementTruck`, etc.
  - → Don't call it `TruckClass` that is tautology just as bad as `TruckImpl` or `ITruck`
- "If you ever will have only one implementation, skip the interface. It creates this naming problem and adds nothing."
- "Hide the implementation behind a static factory method like `Trucks.create()`, for example as an anonymous inner class."

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- Elliptic curves (student UniFR)
  - → ECGroup
  - → ECC (ECElGamalEncryption, ECPedersenCommitment, etc.)
- Zero-Knowledge Proofs
  - → OR-composition (Jürg, Philémon)
  - → Validity proof (Jürg, Philémon)
  - → Batch proofs (Philipp)
  - → Proof of shuffle (Philipp)
  - → ECC proofs
- Secret sharing (Jürg)
  - → Shamir's secret sharing
  - → Verifiable secret sharing
- Symmetric encryption: AES